INSPECTORATUL ŞCOLAR JUDEŢEAN CONSTANŢA

OLIMPIADA DE MATEMATICĂ

Etapa locală – Constanţa, 16.02.2013

Clasa a IX-a

Barem de corectare şi notare

Subiectul 1

$\underbrace{[x]-\left[\frac{1}{x}\right]}_{\in Z}=\{x\}-\left\{\frac{1}{x}\right\}\in(-1,1)\quad(3p)\quad\Rightarrow[x]-\left[\frac{1}{x}\right]=0\ (1p)\Longrightarrow\{x\}=\left\{\frac{1}{x}\right\}\quad(1p),$

iar prin sumare $x=\frac{1}{x}\Rightarrow\quad x^2=1\Rightarrow x=\pm1$, care verifică ecuaţia. (2p)

Subiectul 2

$\overrightarrow{OG_A}=\frac{1}{3}\left(\overrightarrow{OM}+\overrightarrow{OB}+\overrightarrow{OC}\right)\Leftrightarrow\overrightarrow{MG_A}=\frac{1}{3}\left(\overrightarrow{MB}+\overrightarrow{MC}\right)$ şi analoagele (2p); $\overrightarrow{AG_A}=\overrightarrow{AM}+\overrightarrow{MG_A}$ şi analoagele (2p) . Sumând obţinem: $\sum\overrightarrow{AG_A}=\sum\overrightarrow{AM}+\sum\overrightarrow{MG_A}=\sum\overrightarrow{AM}+\frac{1}{3}\sum\left(\overrightarrow{MB}+\overrightarrow{MC}\right)=\sum\overrightarrow{AM}-\frac{2}{3}\sum\overrightarrow{AM}=\frac{1}{3}\sum\overrightarrow{AM}=\vec{0}\Rightarrow\sum\overrightarrow{AM}=\vec{0}$ (2p). Dar $\sum\overrightarrow{AM}=\sum\left(\overrightarrow{AG}+\overrightarrow{GM}\right)=\underbrace{\sum\overrightarrow{AG}}_{\vec{0}}+3\overrightarrow{GM}=\vec{0}\Rightarrow G=M$ (1p).

Subiectul 3

$\frac{a+b}{a+c}=\frac{1}{1+\frac{c}{a}}+\frac{1}{\frac{a}{b}+\frac{c}{b}}\le\frac{1+\frac{a}{c}}{4}+\frac{\frac{b}{a}+\frac{b}{c}}{4}$ (inegalitatea dintre media aritmetică şi cea armonică) şi analoagele (3p). Prin sumare : $\sum\frac{a+b}{a+c}\le\frac{1}{4}\left(3+\frac{a}{c}+\frac{b}{a}+\frac{b}{c}+\frac{b}{a}+\frac{c}{b}+\frac{c}{a}+\frac{c}{b}+\frac{a}{c}+\frac{a}{b}\right)\le\frac{1}{4}\left(3+\frac{b^2}{a^2}+\frac{c^2}{b^2}+\frac{a^2}{c^2}+2\frac{a}{c}+2\frac{b}{a}+2\frac{c}{b}\right)\Leftrightarrow\frac{a}{b}+\frac{c}{a}+\frac{b}{c}\le\frac{b^2}{a^2}+\frac{c^2}{b^2}+\frac{a^2}{c^2}$ (2p). Notăm $x=\frac{b}{a}$, $y=\frac{a}{c}$, $z=\frac{c}{b}\Rightarrow\frac{b}{c}=xy,\quad\frac{c}{a}=xz,\ \frac{a}{b}=yz,$ deci $x^2+y^2+z^2\ge xy+xz+yz\ (A)\quad(2p)$.

Subiectul 4

$n=1:\ a_2=\frac{2}{4}\left(\frac{1}{6}+\frac{1}{2}\right)=\frac{1}{2}\cdot\frac{4}{6}=\frac{2}{6}$

$n=2:\ a_3=\frac{3}{5}\left(\frac{2}{6}+\frac{1}{2}\right)=\frac{3}{5}\cdot\frac{5}{6}=\frac{3}{6}$ (2p)

P(n): $a_n=\frac{n}{6}$

P(1): $a_1=\frac{1}{6}$ (A)

....

P(n+1): $a_{n+1}=\frac{n+1}{6}$

$a_{n+1}=\frac{n+1}{n+3}\left(\frac{n}{6}+\frac{1}{2}\right)=\frac{n+1}{n+3}\cdot\frac{n+3}{6}=\frac{n+1}{6}$ (4p).

Deci $a_{2000}=\frac{2000}{6}=\frac{1000}{3}$ (1p).

Observaţie: Orice altă soluţie corectă se notează cu maxim de puncte